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REMARKS ABOUT THE INFLUENCE OF DENSITY FLUCTUATIONS IN TURBULENT BOUNDARY LAYERS FOR COMPRESSIBLE FLOW

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16. Abstract The contributions of the density fluctuations to the momentum and heat transport near the wall within the boundary layer are investigated in a more rigorous fashion than was done in a paper by Szablewski. Relationships between temperature, pressure and density fluctuations are presented as well as density fluctuation terms in the wall flow law.		
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REMARKS ABOUT THE INFLUENCE OF DENSITY FLUCTUATIONS IN TURBULENT BOUNDARY LAYERS FOR COMPRESSIBLE FLOW

J. C. Rotta*

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1. INTRODUCTION

The problem of the effects of density fluctuations in turbulent boundary layers under compressible flow conditions has been discussed many times. It is interesting because the density fluctuations appear as an additional degree of freedom of the turbulent motion if the restricting conditions for the incompressible nature of the flow medium is dropped. In a theoretical investigation [1] of the influence of Mach number and heat transfer on the flow in the vicinity of the wall, we were careful to carry along all terms containing density fluctuations in the mathematical treatment. In the subsequent calculations based on semi-empirical formulas,it was not possible to take them into account. Recently W. Szablewski [2] investigated the question again, also for flow near the wall.

In this paper we will point out the reasons for the divergence in results of the two publications. We will also discuss a few relationships which are useful for estimating the density fluctuations. After this we will again investigate the contributions of the density fluctuations to the momentum and heat transport near the wall, because since the publication of my original paper [1], additional test results have become

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available.

2. MOMENTUM AND HEAT TRANSPORT NEAR THE WALL.

Szablewski uses the idea, which is plausible, that the mass flux produced by the statistical correlation between the velocity and density fluctuations during turbulent momentum and heat transport is important, where ϱ' are the density fluctuations and v' are the velocity fluctuations perpendicular to the wall. The estimation used for the turbulent mass flux using an idea of the Boussinesq or Schmidt type is certainly justified, as long as such exchange models are considered to be acceptable. Also, the assumption that the exchange variables for temperature and mass transport are the same but different for momentum transport seems to be reasonable according to our present knowledge about turbulent mixing processes. However, it is necessary to accurately consider the turbulent mass flux in the mass conservation law.

In the region very close to the wall, the total mass flux perpendicular to the wall must vanish provided the wall is impermeable:

$$\overline{\varrho v} = 0 . \tag{1}$$

This is found by integrating the continuity equation, because the derivatives of all time averages with respect to the coordinates parallel to the wall can be ignored for small wall separations. If we split these quantities into time averages and fluctuation values

$$v = \bar{v} + v', \quad \varrho = \bar{\varrho} + \varrho' \text{ etc.} \tag{2}$$

then from (1) we find

$$\bar{\varrho}\,\bar{v} + \overline{\varrho' v'} = 0 . \tag{3}$$

If this relationship is introduced in the relationship for momentum transport, then the sum of the first and third terms on the right side of the Equation (5) in Szablewski's paper becomes zero. This means that we only have

$$\overline{\varrho u v} \approx \bar{\varrho}\,\overline{u'v'} \tag{4}$$

In a similar way, in Equation (8) of the same paper the first term is compensated by the third term on the right side, so that for the convective heat transport perpendicular to the wall, we find

$$c_p \overline{\varrho v T} \approx c_p \bar{\varrho}\,\overline{v'T'} \tag{5}$$

This means that in both relationships the terms containing the density fluctuations drop out, which was the topic of Szablewski's investigation*. The density fluctuations only occur in those terms which were ignored by Szablewski because they were negligibly small. The author's work has led to the following exact relationships /188

$$\overline{\varrho u v} = \bar{\varrho}\,\overline{u'v'} - \overline{\varrho' v'}\,\frac{\overline{\varrho' u'}}{\bar{\varrho}} + \overline{\varrho' u' v'}\,, \tag{6}$$

$$c_p \overline{\varrho v T} = c_p\left(\bar{\varrho}\,\overline{v'T'} - \overline{\varrho' v'}\,\frac{\overline{\varrho' T'}}{\bar{\varrho}} + \overline{\varrho' v' T'}\right). \tag{7}$$

The estimation of all density fluctuation terms is not simple, because only the term $\overline{\varrho' v'}$ can be determined using the exchange theorem. It cannot be used for the correlations between ϱ' and u' or ϱ' and T'. This does not mean that such correlations do not exist. For moderate Mach numbers and heat transfer coefficients, the density fluctuation terms in (6) and (7)

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* This statement not only holds for the special assumptions of a region near the wall, but for the entire boundary layer; see Appendix.

remain small with respect to $\bar{\varrho}\,\overline{u'v'}$ and $\bar{\varrho}\,\overline{v'T'}$. However, this is certainly not true for arbitrarily large Mach numbers.

3. RELATIONSHIPS BETWEEN THE TEMPERATURE, DENSITY AND PRESSURE FLUCTUATIONS

In the case of turbulent flows of gases, there are fluctuations in the three velocity components, in the pressure, in the density and temperature. In order to deal with the equations and to estimate the terms, it is very useful if one can reduce the number of fluctuation variables. It is relatively easy to eliminate the density fluctuations using the equation of state for gases. This leads to a few interesting relationships. The following discussion is first of all valid in general for turbulent gas flows and assumes that the equation of state for ideal gases

$$p = R\varrho T \tag{8}$$

is applicable, where R is the gas constant. By forming the average of (8) we find

$$\bar{p} = R\,\overline{\varrho T} = R(\bar{\varrho}\,\bar{T} + \overline{\varrho' T'}). \tag{9}$$

From this it is possible to eliminate R in (8) and we find

$$\left(1 + \frac{\overline{\varrho' T'}}{\bar{\varrho}\,\bar{T}}\right)\frac{p}{\bar{p}} = \frac{\varrho T}{\bar{\varrho}\,\bar{T}}. \tag{10}$$

After separation of the average value and the fluctuation variable according to (2), it is possible to express the density fluctuations in (10) by means of

$$\frac{\varrho'}{\bar{\varrho}} = \left(1 + \frac{\overline{\varrho' T'}}{\bar{\varrho}\,\bar{T}}\right)\frac{p'}{\bar{p}} - \frac{T'}{\bar{T}} - \frac{\varrho' T' - \overline{\varrho' T'}}{\bar{\varrho}\,\bar{T}} \tag{11}$$

If we assume that the fluctuations are small compared with the average values, it is possible to first ignore products of fluctuation variables. Therefore, we obtain the following first order approximation

$$\frac{\varrho'}{\bar{\varrho}} \approx \frac{p'}{\bar{p}} - \frac{T'}{\bar{T}}. \tag{12}$$

One can then determine the fluctuation products in (11) and we obtain the second order approximation in the following form

$$\frac{\varrho'}{\bar{\varrho}} = \left(1 + \frac{\overline{p'T'}}{\bar{p}\,\bar{T}} - \frac{\overline{T'^2}}{\bar{T}^2}\right)\frac{p'}{\bar{p}} - \frac{T'}{\bar{T}} - \frac{p'\,T' - \overline{p'T'}}{\bar{p}\,\bar{T}} + \frac{T'^2 - \overline{T'^2}}{\bar{T}^2}. \tag{13}$$

This iteration method can be continued up to any arbitrary accuracy. It converges as long as $p'/\bar{p} < 1$ and $T'/\bar{T} < 1$, which is certainly always satisfied.

We will now discuss the mass flux $\overline{\varrho' v'}$ caused by turbulent motion and we will only consider the first approximation (12). We then find

$$\frac{\overline{\varrho' v'}}{\bar{\varrho}} = -\frac{\overline{v' T'}}{\bar{T}} + \frac{\overline{p' v'}}{\bar{p}}. \tag{14}$$

It is interesting to compare this result with the estimations made by Szablewski according to the exchange theorem. /189
From the general relationship $\bar{\varrho}\,\bar{T} =$ const. which is valid for boundary layers we find

$$\frac{\partial\bar{\varrho}/dy}{\bar{\varrho}} = -\frac{\partial\bar{T}/\partial y}{\bar{T}}. \tag{15}$$

Together with $\overline{\varrho' v'} = -\varepsilon(\partial\bar{\varrho}/\partial y)$ and $\overline{v' T'} = -\varepsilon(\partial\bar{T}/\partial y)$, we find

$$\frac{\overline{\varrho' v'}}{\bar{\varrho}} = -\frac{\overline{v' T'}}{\bar{T}}. \tag{16}$$

The quantity ε is the exchange variable. Equation (16) differs from Equation (14) because there is no second term. The average product of the pressure and velocity fluctuations $\overline{p' v'}$ which occurs in (14) also occurs in the expression for the kinetic fluctuation energy and there it is interpreted as the energy diffusion caused by pressure fluctuations. The second term on the right side of (14) is small with respect to the first term in many cases, so that (16) is often a good approximation. However, it can be seen that there are other influences in the case of the turbulent mass flux and it is not possible to state how large the error will be in any individual case if the pressure fluctuations are ignored.

4. ESTIMATION OF THE DENSITY FLUCTUATION TERMS FOR THE WALL LAW.

Using relations (12) or (13), it is possible to make estimations in cases which cannot be found using the exchange theorem. For example, we have the terms which occur in (6) and (7). In order to not have to write down a large number of terms, we will not consider the pressure fluctuations here and we find

$$\frac{\overline{\varrho' u'}}{\bar{\varrho}} \approx -\frac{\overline{u' T'}}{\bar{\varrho}}, \qquad \frac{\overline{\varrho' T'}}{\bar{\varrho}} \approx -\frac{\overline{T'^2}}{\bar{T}}, \qquad \frac{\overline{\varrho' u' v'}}{\bar{\varrho}} \approx -\frac{\overline{u' v' T'}}{\bar{T}}, \qquad \frac{\overline{\varrho' v' T'}}{\bar{\varrho}} \approx -\frac{\overline{v' T'^2}}{\bar{T}}. \tag{17}$$

By introducing correlation coefficients defined by

$$R_{uv} = \frac{\overline{u'v'}}{(\overline{u'^2}\,\overline{v'^2})^{1/2}} \quad \text{etc.} \tag{18}$$

Equations (6) and (7) are written in the following clear form using (17).

$$\overline{\varrho u v} = \bar{\varrho}\,\overline{u' v'}\left[1 - \frac{R_{vT} R_{uT}}{R_{uv}} \frac{\overline{T'^2}}{\bar{T}^2} - \frac{R_{uvT}}{R_{uv}} \frac{(\overline{T'^2})^{1/2}}{\bar{T}}\right], \tag{19}$$

$$c_p \overline{\varrho v T} = c_p \bar{\varrho}\,\overline{v' T'}\left[1 - \frac{\overline{T'^2}}{\bar{T}^2} - \frac{R_{vTT}}{R_{vT}} \frac{(\overline{T'^2})^{1/2}}{\bar{T}}\right] \tag{20}$$

As can be seen, the third terms in the bracket of (6) and (7) are proportional to the temperature fluctuations $(\overline{T'^2})^{1/2}$. The second term increases with $\overline{T'^2}$.

The fact that, on the one hand, u' and on the other hand, T' are statistically correlated with v' means that it is likely that a correlation between u' and T' exists. Hot wire measurements in the boundary layer for compressible [3] and for incompressible boundary layers over heated walls [4] justify this assumption. We found correlation coefficients $R_{uT} = \overline{u'T'}/(\overline{u'^2}\,\overline{T'^2})^{1/2}$ on the order of 0.8. The sign is controlled by the sign of $R_{uv} R_{vT}$. The quotient $R_{vT} R_{uT}/R_{uv}$ is therefore always positive and has the magnitude ~ 0.8, because R_{vT} and R_{uv} have about equal magnitudes. D. S. Johnson [4] also experimentally determined the triple correlation coefficient $R_{vTT} = \overline{v'T'^2}/[(\overline{v'^2})^{1/2}\,\overline{T'^2}]$ in experiments. R_{vTT} varies between +0.1 and -0.7. It is not even possible to give a rough estimation of the ratio R_{vTT}/R_{vT} in (20). Since R_{vT} has a magnitude of about 0.5, we must expect that R_{vTT}/R_{vT} has a magnitude of one or more. No measurements regarding the correlation coefficient R_{uvT} are known. Because of the strong correlation between u' and T', the relationship

$$\frac{R_{uvT}}{R_{uv}} \approx \frac{R_{vTT}}{R_{vT}} \tag{21}$$

is reasonable. As far as is known at this time, the correlation coefficients are not noticeably influenced by the Mach number. /190

The temperature fluctuations will be approximately proportional to the gradient of average temperature $\partial\bar{T}/\partial y$. Using the similarity relationships of the wall law [5] we derive *

$$\frac{(\overline{T'^2})^{1/2}}{\bar{T}} \sim M_\tau^2 \tag{22}$$

where $M_\tau = \sqrt{c_f/2}\, Ma_\infty$, where c_f is the local friction coefficient, and Ma_∞ is the Mach number of the flow outside of the boundary layer. The experiments of Kistler result in the following for the region near the wall

$$\frac{(\overline{T'^2})^{1/2}}{\bar{T} M_\tau^2} \approx 10. \tag{23}$$

The quantity M_τ increases with Ma_∞ but not in an unlimited fashion, because the friction coefficient c_f at the same time decreases. We do not believe that the value $M_\tau = 0{,}2$ is exceeded.

* See footnote at beginning of article.

This means that values such as $(\overline{T'^2})^{1/2}/\bar{T} \sim 0{,}4$ are not to be excluded in extreme cases; in this case $\overline{T'^2}/\overline{T}^2$ would be $\sim 0{,}16$. For very large Ma_∞ , on the other hand, the pressure fluctuations can no longer be ignored. Apparently these have a reducing effect on the density fluctuation terms.

The last statements were very vague. Because of the possible order of magnitude of the density fluctuation terms, it would be very desirable to have further clarification regarding the pressure and temperature fluctuations. However, it is important to note that the expressions in brackets in (19) and (20) have almost the same values in all cases, so that it hardly seems necessary to fear a strong effect on the relationship between the temperature and velocity distribution. This assumption was already discussed in my earlier paper.

APPENDIX

The terms $\bar{u}(-\overline{\varrho' v'})$ and $T(-\overline{\varrho' v'})$ which formally occur in the momentum and energy equation vanish because of the continuity equation even in cases where one does not intend to use a restriction to regions near the wall. In order to show this using the example of the momentum equation, we will, as a first step, integrate the continuity equation (3) in the paper of Szablewski [2]. For impermeable surfaces ($\bar{v} = v' = 0$ for y = 0), we find

$$\frac{\partial}{\partial x}\int_0^y \bar{\varrho}\,\bar{u}\,dy' + \bar{\varrho}\,\bar{v} = (-\overline{\varrho' v'})\,. \tag{A,1}$$

By integrating the momentum Equation (6) in the same paper we find

$$\frac{\partial}{\partial x}\int_0^y \bar{\varrho}\,\bar{u}^2\,dy' + \bar{\varrho}\,\bar{u}\,\bar{v} = -y\frac{d\bar{p}}{dx} + \left(\overline{\mu\frac{\partial u}{\partial y}}\right) + \bar{\varrho}(-\overline{u'v'}) + \bar{u}(-\overline{\varrho' v'}) + C\,. \tag{A,2}$$

The magnitude of the integration constant C is determined from the boundary condition at y = 0

$$C = -\left(\mu \frac{\partial u}{\partial y}\right)_0 = -\tau_0 , \tag{A,3}$$

where τ_0 is the wall shear stress. If we now substract Equation (A,1) multiplied by $\bar{u}$ from the relationship (A,2), we finally obtain the following using Equation (A,3)

$$\frac{\partial}{\partial x}\int_0^y \bar{\varrho}\,\bar{u}^2\,dy' - \bar{u}\,\frac{\partial}{\partial x}\int_0^y \bar{\varrho}\,\bar{u}\,\partial y' = -y\,\frac{d\bar{p}}{dx} + \left(\mu\,\frac{\partial u}{\partial y}\right) + \bar{\varrho}(-\,\overline{u'v'}) - \tau_0 \,. \tag{A,4}$$

This equation is exact in a sense of the initial equations given by Szablewski and applies for the entire boundary layer. The density fluctuations do not occur in them.

REFERENCES

1. Rotta, J. Z. Flugwiss, Vol. 7, 1959, p. 264.
2. Szablewski, W. Ing.-Arch., Vol. 30, 1961, p. 317.
3. Kistler, A. Phys. Fluids, Vol. 2, 1959, p. 290.
4. Johnson, D. S. J. Appl. Mech., Vol. 26, 1959, p. 325.

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